STAT C-601
DESIGN OF EXPERIMENTS

Design of Experiments Question Bank

1 What are uniformity trials? Discuss their utility in Design of Experiments.

2 In a randomized block design there are only two blocks. Let k be the number of treatments and $\overline{x_1}$ and $\overline{x_2}$ be the average yields of two blocks show that the between blocks sum of squares can be expressed as $\frac{k}{2}\left(\overline{x_1}-\overline{x_2}\right)^2$.

3 Discuss how the efficiency of an experiment can be increased by using local control and by increasing the number of replications. Determine its efficiency of LSD relative to RBD taking columns as blocks and CRD.

4 How do the size and shape of the plots and blocks affect the results of field experiments? Explain.

5 Derive the ANOVA table for an L.S.D. How would you test:

(i) The hypothesis of equality of all treatment effects; and

(ii) The hypothesis of equality of two specific treatment effects?

6 Derive the standard error of the difference between two estimated treatment means, one of which involves the missing plot, for a LSD with a single missing observation.

7 What is meant by missing plot technique? Show how to estimate a missing value in a RBD. Calculate the bias involved in the estimated value and hence the ANOVA?

8 Derive the standard error of the difference between two estimated treatment means, one of which involves the missing plot, for a RBD with a single missing observation.

9 What is meant by missing plot technique? Show how to estimate a missing value in a LSD. Calculate the bias involved in the estimated value and hence the ANOVA?

10 Define a BIBD with parameters v, b, r, k and λ. State and prove the relationships among its parameters.

11 For a BIBD with parameters v, b, r, k and λ show that the adjusted treatment sum of squares is $\frac{k}{v\lambda}\sum_{i=1}^{v} Q_1^2$ where Q_i is the adjusted total for the ith treatment. Also compute the degrees of freedom.

12 If N is the incidence matrix of a symmetric BIBD show that

$$(NN')^{-1} = \frac{1}{r-\lambda}\left(I_v - \frac{\lambda}{r^2}E_{vv}\right)$$

Hence show that any two blocks of the symmetric BIBD have λ treatments in common.

13 Derive the efficiency of BIBD relative to RBD.

14 For a BIBD compute $V(\hat{\alpha}_i - \hat{\alpha}_l)$.

15 Define a resolvable BIBD and Affine resolvable BIBD. Also for a resolvable BIBD show that $b \geq v + r - 1$. Discuss the efficiency of BIBD as compared to RBD.

16 The following are three key-blocks (Principal Block) of a layout plan before randomization for a 2^4 experiment with factors A, B, C and D:

Replication I: (1), *abc, abd, cd*

Replication II: (1), *abc, acd, bd*

Replication III: (1), *abd, acd, bc*

Find out the effect or effects confounded and construct the analysis of variance table for

17 Construct a 2^5 FE, the confounded effects being ABE and BCD.

18 If some of the elements of the key block of a 2^5 factorial experiment are (1), ad, bc, abe, then identify all the confounded effects. Write down the contents of all the blocks.

19 Given one block of a 2^4 factorial experiment with factors A, B, C and D, identify the confounded effect:

Block: *c, a, d, acd, be, ab, bd, abcd*

20 Key block of a 2^4 factorial experiment with factors A, B, C and D is given as:

Block: 0000, 1100, 0011, 1111

Identify the confounded effects.

21 Construct a 2^5 factorial experiment with factors A, B, C, D and E in 2^2 blocks in two replicates, confounding ABC and BDE in one replicate and BE and ACE in the other replicate. Identify if any other effect is also confounded in each replicate.

22 Show that in a 2^n factorial experiment for $2 \leq n \leq 5$, replicated r times in randomized blocks of 2^k plots, we obtain $r\ 2^{n-2}/\sigma^2$ units of information on every maineffect and interaction.

23 What is confounding? Construct a completely confounded 2^3 factorial experiment in blocks of size 4 with 8 blocks, so that it completely confounds the second order interaction. How the arrangement can be modified so as to recover at least partial information about the 2 and 3 factor interaction components and complete information

about all the main effects. Explain how Yates algorithm can be used to compute the total effects, main effects and sum of squares due to various effects.

24 The following are two blocks of a layout plan before randomization for a 3^3 factorial experiment with factors A, B, and C:

Rep. 1: $a, a^2c^2, c, a^2bc, b, abc^2, b^2c^2, a^2b^2, ab^2c$

Rep II: $a, abc, a\ b^2c^2, a^2c, a^2bc^2, a^2b^2, c^2, b, b^2c$

Identify the confounded effects in each replication.

26 A 3^3 factorial expt. with factors A, B, and C is arranged in two replicates of 3 blocks each. If sime of the elements of one of the blocks of each replicate are given as

Rep 1: 000, 101, 012

Rep 2: 000, 102, 111

write down the contents of the remaining blocks. Also list all the confounded effects.

27 Explain Yates technique for calculating sum of squares due to main effects and interaction effects. Also write down the ANOVA table in case of a 3^2 factorial experiment with *r* replications.

28 What is meant by confounding in factorial experiments? Distinguish between partial and total confounding. Construct a system of partial confounding for a 3^2 factorial experiment in blocks of size 3 with 6 blocks, so that atleast partial information can be obtained about two factor interaction components and full information about the main effects.

29 The following is the block of a layout plan before randomization for a 3^3 factorial experiment with factors A, B, and C:

Rep. I: $(1), ac, bc^2$

Identify all the confounded effects. Write down the contents of the remaining blocks.

30 Construct one replication of a confounded 2^5 factorial experiment using blocks of size 2^3, such that no main effect and two factor interactions are confounded.

STAT C-602
MULTIVARIATE ANALYSIS AND NONPARAMETRIC METHOD

Multivariate Analysis and Non Parametric Methods

1) Describe Wald 's S.P.R.T. , its O.C. and A.S.N. functions .

2) Construct S.P.R.T. for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(0 < \theta_0 < \theta_1)$ on the basis of a random sample drawn from a Poisson distribution with parameter θ. Also obtain its O.C. and A.S.N. functions.) .

3) For the S.P.R.T. of strength (α_1 , β_1) $and\ for\ given\ A = \frac{1-\beta}{\alpha}$ $and\ B = \frac{\beta}{1-\alpha}$,

prove that

(i) $\alpha_1 \leq \frac{\alpha}{1-\beta}$, $\beta_1 \leq \frac{\beta}{1-\alpha}$,

(ii) $\alpha_1 + \beta_1 \leq \alpha + \beta$.

4) Construct S.P.R.T. for testing $H_0 : \theta = \theta_0$ against $H_1 : \theta = \theta_1$ $(0 < \theta_0 < \theta_1)$ on the basis of a random sample drawn from a population with p. d. f.:

$f(x,\theta) = \theta e^{-\theta x}, x > 0, \theta > 0.$

Also obtain its O.C. and A.S.N. functions.

5) If S.P.R.T. of strength (α,β) terminates with probability one then determine its stopping bounds.

6) What are the advantages and disadvantages of non parametric tests?

7) Discuss the Mann-Whitney-Wilcoxon test for testing whether two samples are drawn from the same continuous population. How is the test carried out for large samples? Also discuss the case of ties.

8) Discuss the Kruskal-Wallis test for testing the null hypothesis that K- samples come from same continuous population. Explain the large sample behaviour of the test.

9)Name a non-parametric test which is alternative to the one-way analysis of variance for testing that K independent samples are drawn from same continuous population. Describe its method and function in detail.

10) Describe the sign test, stating clearly the assumptions involved.

11) Define a run and the length of a run . Describe the Run test in detail for testing the equality of the two populations. How is the test carried out for large samples? Also discuss the case of ties ?

12) Let X be a 3-dimentional random vector, i.e., $X=(X_1, X_2, X_3)^T$ and distributed as $N_3 \sim (\mu, \sum)$ with $\mu=(-3, 1, 4)^T$ and $\sum=\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$. Find out the conditional distribution of X_1 given $(X_2 X_3)=(x_2, x_3)$.

13) Let X be a 3-dimentional random vector, i.e., $X=(X_1, X_2, X_3)^T$ have the covariance matrix $\sum=\begin{bmatrix} 1 & -2 & 0 \\ -2 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$.

(i). Determine the principal components of Y_1, Y_2 and Y_3, (ii). Show that the principal components are uncorrelated and have variances equal to the eigen values of $\sum$, (iii). Show that the total population variance is equal to the sum of variances of principal components, (iv). Find the proportion of the total variance due to the first two principal components and comment and (v). Find the correlation coefficients r (Y_1, X_1) and r (Y_1, X_2).

14) Let (X, Y) follow BND with pdf $f(x, y) = \boldsymbol{k} e^{[-4x^2-6xy-9y^2]}$. Find (i). The value of k
(ii). Correlation coefficient between X and Y, (iii). Marginal distribution of Y and
(iv). Conditional distribution of Y given X=x.

15) Consider the factor model
$X_1-\mu_1= 4F_1+ F_2+\varepsilon_1$, $X_2-\mu_2= 7F_1+ 2F_2+\varepsilon_2$, $X_3-\mu_3= -F_1+ 6F_2+\varepsilon_3$ and $X_4-\mu_4= F_1+ 8F_2+\varepsilon_4$,
With cov $(\varepsilon)=\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix}$. (i). Compute the covariance matrix $\sum$, (ii). Find communalities h_i^2, i=1, 2, 3 and (iii). Show decomposition of variance into communality and specific variance

16) If $\underset{\sim}{X} \sim N_p (\mu, \sum)$ then show that the marginal distribution of any set of components of $\underset{\sim}{X}$ is multivariate normal with mean, variance and covariance obtained by taking the corresponding components of μ and $\sum$ respectively.

17) Show that if $\underset{\sim}{X} \sim N_p (\mu, \sum)$ then $Y = C \underset{\sim}{X} \sim N_p (\mu, C \sum C^t)$ where C is a non singular matrix.

18) Explain the concept of multiple and partial correlation coefficients .Find te equation of plane of regression of X_1 on X_2 and X_3.

19) Define multivariate normal density for p variables. Also define mean vector and variance covariance matrix.

20) If (X, Y) ~ $\mathcal{BVN}$ $(\mu_1 , \mu_2 , \sigma_1^2, \sigma_2^2 , \rho)$ then show that linear combination of X and Y is a normal variate . Prove that converse also holds good.

21) If (X, Y) ~ $\mathcal{BVN}$ $(\mu_1 , \mu_2 , \sigma_1^2, \sigma_2^2 , \rho)$ then find the correlation coefficient between e^x and $e^{y.}$

22) If Xhas a p-variate normal distribution $N_p (\mu , \sum)$ and $X= (X_1 , X_2)$ where X_1 is a qx1 sub vector then obtain conditional distribution of X_1 given $X_2 = x_2$. Hence show that the mean of X_1 given $X_2 = x_2$ is a linear function of X_2 and that covariance matrix of X_1 given $X_2 = x_2$ does not depend on X_2,

23) Explain

1) Discriminant Analysis

2) Principal Component Analysis

3) Factor Analysis

DSE-3(B)
SURVIVAL ANALYSIS AND BIOSTATISTICS

B. Sc. (H) Statistics Sem VI

DSE Paper : STAT-DSE-3(B)

Name of the Paper : Survival analysis and Bio-Statistics

1. Define Bathtub type survival model. Obtain its hazard function, survival function and death density function.

2. Define type-II censoring. Under this censoring estimate mean survival time assuming that duration of disease follows exponential distribution.

3. Briefly explain two non- parametric method for defining survival function. Using either of the method estimate the survival function also compute the variance of the estimate for survival function.

4. Suppose that two risks R_δ and $R\varepsilon$ ($\delta \neq \varepsilon$) are operating in the population such that $Q_{i\delta} > Q_{i\varepsilon}$. .Show that

(1) $q_{i\delta} > q_{i\varepsilon}$

(2) $q_{i.\delta} < q_{i.\varepsilon}$

5. What are different phases of clinical trials? Write the objectives of all the three phases of the clinical trials.

6. Define simple epidemic model. Find the probability of two individuals getting infection given that there are n susceptible individuals at the beginning.

7. Define crude, net probability type A, net probability type B, and partially crude probability of death and show that

$$Q_{i\,\delta\,.\varepsilon} = Q_{i\,\delta}[1 - p_i^{(q_i - Q_{i\,\varepsilon})/q_i}]/(q_i - Q_{i\,\varepsilon})$$

8. Define modified chi-square method. Under this method find the estimators of $Q_{i\delta}$ and p_i.

9. Define type –I censoring. In a clinical trial, 10 patients with advanced cancer of prostrate survived (in months) as follows

7, 19, 13, 12, 21, 16, 4, 9, 21, 11

Assuming the exponential survival distribution function for these patients, compute the mean survival time and survival rate estimates. Also, estimate mean survival time and its variance, if the study is terminated after 15 months.

10. Define death density, survival function and hazard function when T follows Weibull distribution. Obtain mean survival time and variation in survival time when scale parameter=0.5 and shape parameter=1.5. Also comment on the nature of hazard function for given parameters.

11. Suppose that the survival distribution of a group of patients follows the exponential distribution with parameter 0.65. Find

i) The mean and variance survival time.

ii) The probability of surviving at least 1.5 years.

12. For the following data

Year after diagnosis	*Alive at the beginning of the Interval*	*Died during the interval*	*Lost to follow-up*	*Withdrawn alive during the interval*
0-1	126	47	4	15
1-2	60	5	6	11
2-3	38	2	-	15
3-4	21	2	2	7
4-5	10	-	-	6

Compute est($S(t_i)$) and est (var($S(t_i)$)) for each interval.

13. What is duration of an epidemic? Obtain mean duration of an epidemic under simple stochastic epidemic model.

14. Consider the following survival data. Compute survival function, probability density function and hazard function:

Year of follow-up	No Alive at the beginning of the interval	No dying in the interval
0-1	1000	245
1-2	755	182
2-3	573	174
3-4	399	128
4-5	271	76
5-6	195	60
6-7	135	43

15. Suppose that k risks of death are operating independently in a population. Let λ_i be the hazard rate associated with the i^{th} risk and $\lambda = \sum_{i=1}^{k} \lambda_i$ be the total force of mortality, then; show that the probability of dying due to i^{th} cause will be λ_i / λ.

16. What do you mean by an epidemic? Define simple stochastic epidemic model. Obtain the probability of no susceptible getting infected till time t in a simple epidemic model. If an epidemic is initiated by 10,000 susceptible and 1 infective with infection rate 0.002, then determine the probability of no susceptible getting infected till 8 units of time.

17. Use method of maximum likelihood to estimate crude, net and partially crude probability of death.

18. Define crude probability and partially crude probability of death. Establish the interrelationship between the two.

19. Obtain the expression for death density function due to risk R_i, when the

i) Competing risks are independent.
ii) Competing risks are dependent.

20. Define survival function, death density function and hazard function. Find

(1) Interrelationship between above functions.
(2) F(t) and h(t) when $S(t)= \exp(-t^r)$

21. Explain all the phases of clinical drug trials.

(b) Define simple and general stochastic epidemic models. Find the duration of the epidemic under simple epidemic model where initially there are n susceptible and one infective.

22. Consider the following tumor free time in days of the 10 rats on a low fat diet. Calculate Kaplan Meier estimates of S(t) for all rats and standard error of S(t) for every uncensored observation.

Rat No	**4**	**6**	**1**	**7**	**8**	**5**	**3**	**9**	**2**	**10**
Tumor Free time	50	56	65	66	73	77	84	86	87	119

23. Consider the following two Gamma distribution as survival models;

(i)Scale parameter =1, shape parameter=0.5
(ii)Scale parameter =0.5, shape parameter=2

DSE-4(A)
FINANCIAL STATISTICS

FINANCIAL STATISTICS QUESTIONS

Forward contract. Future Price . price of call and Put Option. hedging

Q1. A long forward contract on a 5 year bond currently trading at $ 900 . The delivery price is $ 910 . The time to maturity is 1 year. The coupon payment of $ 60 occurs after 6 months and 12 months. The continuously compounded annual interest rate for 6 and 12 months are 9% and 10% respectively. Obtain the value of forward contract and also find the price of forward contract.

Q2. The current interest rate is 7%. Given the opportunity in one of the 3 bonds are listed below. Which should you buy ? Sell short ?

BOND	FACE VALUE	ANNUAL COUPON RATE	MATURITY	PRICE
A	1000	4%	1 YR	990
B	1000	7.50%	17 YR	990
C	1000	8.50%	25 YR	990

Q3. Consider a European call and put option with the same non-dividend paying underlying asset, a strike price of X = 100 USD and maturity in T = 1. The current spot price is S0 = 100 USD, the two option prices are C0 = 14 USD and P0 = 8 USD. Determine the risk-free interest rate r that for which this market is free of arbitrage.

Q4. Spot Price of Dividend Yielding Share is $100. A dividend of 2 months and further $4 per share is expected at the end of 3rd Month. If Risk free Interest Rate is 24% p.a. with Continous Compounding then calculate the Future Price at which 3 Month share should trade.

Q5. Spot Price of a Share of a Company is $ 65 with Exercise Price of $ 60 with time to expiry of 3 months , Risk Free Rate of Return is 10% and Standard Deviation of return of share is 20% .Calculate the price of call option. And show how price will change when duration till expiry is 2 months and 1 month. Also plot the premium against time to expiry .

Q6. Spot price of shares of 'X' company is $ 60 with an exercise price of $ 60 with time to expiration is 6 months i.e. 0.5 year , risk free rate of return is 12% i.e. 0.12 and standard deviation of return of share is 30% i.e. 0.30 . calculate Hedge Ratio and interpret it .

Q7. Consider a forward start option which one year call option with strike price equal to the stock price at that time . Given that :

A. European call option on stock pays no dividend

B. Stock volatility is 30 %

C. Forward price for delivery of one share of stock one year from today is 100.

D. continuously compounded risk free rate is 8%.

Compute value of forward option.

Q8. Assuming that on March 6 , 2018 AT & T share is trading at $20.50 per share . Consider a call option with strike price of $20 expiring on 20¬TH July . Using past stock price , the Standard Deviation in the log prices for AT &T is estimated at 60% . There is one dividend amounting to $0.15 and it will be paid in 23 days . The riskless rate is 4.63% . Calculate value of call option.

Q9. On March 6, 2016, for instance, you could have purchased an AT&T call expiring on January 17, 2018. The stock price for AT&T is $20.50 . The following is the valuation of a call option with a strike price of $20. Assume that AT&T's dividend yield will remain 2.51% over this period and that the risk-free rate for a two-year bond is 4.85%. Calculate value of call option.

Q10 . Given S = $ 40 ,σ=30% , r = 8 % . Suppose you sell a 40 strike put with 91 days expiry .

A. Find delta of option

B. If the option is 100 shares . What investment is required for delta hedge portfolio .

Q11. A binomial tree with made every 4 years to calculate the price of one 8 year European option on the stock of company X. The stock price volatility is 0.5 annual continuously compounded risk free rate is 9%. The stock pays no dividend. The put option's strike price is $ 56 and the stock price is $ 70. Find the price of such put option .

Q. (Interest Rate)

Amon wishes to purchase shares of and options contracts on Mythological Industries in order to create a synthetic T-Bill. At a strike price of 23, he sells a call option for 5.43, buys a put for 2.35, and buys the stock for 20. The options expire in one year, and Amon-Ra holds them until expiration, at which time one of them is exercised. Mythological Industries pays no dividends on its stocks. What is the annual continuously compounded rate of return that Amon-Ra earns on his investment?

Q. (Arbitrage)

Spurious, Inc., stock currently trades for $63 per share. Call and put options on Spurious, Inc., stock with strike price of $76 and time to expiration of 17 months are currently available. The call price is $10, and the put price is $19. The annual continuous risk-free rate is 12%, while the annual continuous dividend yield is 7%. Tadart notices that arbitrage profit is possible under these conditions. Calculate the amount of arbitrage profit per share.

Q. (Payoff Diagram)

Consider the option combination strategy known as the strangle. In the strangle strategy a trader buys a put and a call with a different strike price and the same expiration date. The put strike price, K1 is smaller than the call strike price, K2. Draw the payoff table and payoff graph for the strangle strategy.

Q. (Bond Price)

A certain bond issued by Volatile Industries pays annual coupons of $10 for 10 years. The annual effective interest rate is 0.03. A call on the bond with a strike price of $200 expiring in 10 years sells for $20. A put option on the same bond with the same strike price and time to maturity sells for $3. Find the price of the bond.

Q. (Forward Price)

A European call option on impossible-to-open medicine bottles expiring in 17 months has price 45. A European put option on impossible-to-open medicine bottles with the same strike price and expiration date has price 93. Both options have a strike price of 420. The annual continuously compounded interest rate is 0.3445. Find the 17-month forward price of impossible-to-open medicine bottles/

Q. (Put Call Parity)

A zero-coupon bond issued by Indestructible Co. currently sells for $67. The annual effective interest rate is 0.04. A call on the bond with a strike price of $80 expiring in 9 years sells for

$11.56. Find the price of a put option on the bond with the same strike price and time to maturity.

Q. (European Call Option).

Consider a European call option on a stock with current spot price S is $20, dividend is $ 2, exercise price K is 18 and
time to maturity 6 months. The annual risk-free rate r is 10%. What is the upper and lower bound (limit) of the price of the call and put options?

Q. (American Call Option).

Consider an American call option with a 40 USD strike price on a specific stock. Assume that the stock sells for 45 USD a share without dividends. The option sells for 5 USD 1 year before expiration. Describe an arbitrage opportunity, assuming the annual interest rate is 10%.

Q. (Forward Contract)

The price for a forward contract on actuarial textbooks expiring in four months is 700. At a certain strike price, the price of a European call option expiring in four months for actuarial textbooks in 129, while the price of a European put option is 567. The annual continuously compounded interest rate is 0.023. Find the strike price of the call and put options.

Q.(One Period Binomial option)

The stock of Predictable Co. is currently worth $100 per share. In one year, this price can either be $120 or $90. Predictable Co. stock does not pay dividends. The annual continuously compounded risk-free interest rate is 5%. The strike price of a European call option on Predictable Co. stock is $110. Using, the one-period binomial option pricing model, find the price today of one such call option on Predictable Co. stock.

Q. (Cox-Rubinstein binomial tree)

During 54 periods in a binomial model, the stock price of Imperious LLC has gone up 33 times and gone down 21 times. The current price of Imperious LLC stock is $32/share. The stock price volatility is 0.2, and one time period in the binomial model is 6 months. Using a Cox-Rubinstein binomial tree, calculate the original price of Imperious LLC stock.

Q. (HEDGING)

22 days ago, the stock of Vindictive Co. traded for $511 per share. A certain call option on the stock had a delta of 0.66, a gamma of 0.001, and a daily theta of -0.03. The option used to trade for $59. Now the stock trades for $556. The annual continuously compounded risk-free interest rate is 0.08. Find the new option price using the delta-gamma-theta approximation.